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On expansion waves generated by the refraction of a plane shock at a gas interface

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The paper deals with the regular refraction of a plane shock at a gas interface for the particular case where the reflected wave is an expansion fan. Numerical results are presented for the air–CH_4 and air–CO_2 gas combinations which are respectively examples of 'slow–fast' and 'fast–slow' refractions. It is found that a previously unreported condition exists in which the reflected wave solutions may be multi-valued. The hodograph mapping theory predicts a new type of regular–irregular transition for a refraction in this condition. The continuous expansion wave type of irregular refraction is also examined. The existence of this wave system is found to depend on the flow being self-similar. By contrast the expansion wave becomes centred when the flow becomes steady. Transitions within the ordered set of regular solutions are examined and it is shown that they may be either continuous or discontinuous. The continuous types appear to be associated with fixed boundaries and the discontinuous types with movable boundaries. Finally, a number of almost linear relations between the wave strengths are noted.

1. Introduction

An earlier paper (Henderson 1966) discussed the refraction of a shock wave i at the interface between two different gases. It was shown *inter alia* that, when the refraction was regular and the reflected wave a shock r, then the equations of motion describing the flow near the refraction point could be reduced to a single polynomial equation of twelfth degree. It was found that as many as four of the roots could be of physical significance so that the theory was multi-valued to this extent. These roots were arranged as an ordered set by using the strength (pressure ratio) of the transmitted shock t to do the ordering and it was shown that it was the weakest member of the set that agreed with the experimental data obtained by Jahn (1956). Now the reflected wave in a regular refraction may be an expansion e as well as a shock and the ordered set will often be augmented by a single solution of this type. While some discussion was devoted to this solution no detailed numerical data were presented on it. This deficiency is made good here. The data reveal a previously unreported condition in which there exist two alternative solutions of the expansion type. Usually when two such solutions are obtained one of them can be rejected on physical grounds, because, for example, it may require the reflected wave to propagate forward of the refraction

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point. However, in the present case both solutions seem to be physically acceptable. The hodograph mapping technique indicates that the irregular wave system that borders this condition is basically the double Mach reflexion variety. It is argued that this system will have geometrical degeneracy when $M_0 > 4{\cdot}5$. The appearance predicted for this regular–irregular transition is that a 3-wave system containing a reflected expansion of finite strength changes quickly into a 3-wave system containing a reflected shock of finite strength. Transitions between regular solutions within the ordered set are also examined and it is shown that they may occur either continuously or discontinuously. A plausibility argument indicates that the continuous types are associated with fixed boundaries and the discontinuous types with movable boundaries. Finally, it is pointed out that the solutions of the augmented set will have a certain linear character in the relations between the shock strengths.

2. Analysis of the regular refraction

Consider a thin plane membrane of indefinitely small mass which separates two gases that may in general be of different composition but which initially have the same temperature and pressure. Each gas is assumed to occupy a half space of indefinite extent. Suppose that a plane incident shock i begins suddenly in one gas and propagates towards the other. Then it will be refracted at the interface between them and in the process a shock t will be transmitted into the second gas and a reflected wave will be propagated back into the first gas, figure 1. It will be assumed that the entire motion is steady and two-dimensional. When the reflected wave is an expansion fan e the following equations are available from ordinary one-dimensional gasdynamics:

$$\tan\delta_{0,B} = \frac{(P_{1,T}/P_{0,B})-1}{1+\gamma_{0,B}M_{0,B}^2-(P_{1,T}/P_{0,B})} \times\left[\frac{\{2\gamma_{0,B}/(\gamma_{0,B}+1)\}M_{0,B}^2-c_{0,B}-(P_{1,T}/P_{0,B})}{c_{0,B}+(P_{1,T}/P_{0,B})}\right]^{\frac{1}{2}}, \tag{1}$$

$$\nu_1 = c_0^{\frac{1}{2}}[\tan^{-1}\{c_0(M_2^2-1)\}^{\frac{1}{2}}-\tan^{-1}(M_2^2-1)^{\frac{1}{2}} -\tan^{-1}\{c_0(M_1^2-1)\}^{\frac{1}{2}}+\tan^{-1}(M_1^2-1)^{\frac{1}{2}}], \tag{2}$$

$$\left.\begin{aligned}\frac{P_1/P_0+c_0^{-1}}{P_0/P_1+c_0^{-1}} &= \frac{1+\frac{1}{2}(\gamma_0-1)M_0^2}{1+\frac{1}{2}(\gamma_0-1)M_1^2},\\ c_{0,B} &= \frac{\gamma_{0,B}-1}{\gamma_{0,B}+1},\end{aligned}\right\} \tag{3}$$

$$\frac{P_2}{P_1} = \left[\frac{1+\frac{1}{2}(\gamma_0-1)M_1^2}{1+\frac{1}{2}(\gamma_0-1)M_2^2}\right]^{\gamma_0/(\gamma_0-1)}. \tag{4}$$

In the vicinity of the refraction point the following continuity conditions are assumed to apply:

$$P_T/P_B = (P_1/P_0)(P_2/P_1) \tag{5}$$

and

$$\delta_B = \delta_0+\nu_1; \tag{6}$$

and one also has that

$$P_0 = P_B.$$

Under equilibrium conditions the velocities of the incident and transmitted shocks will be equal along the interface and therefore $v_0 = v_B$. The speed of sound

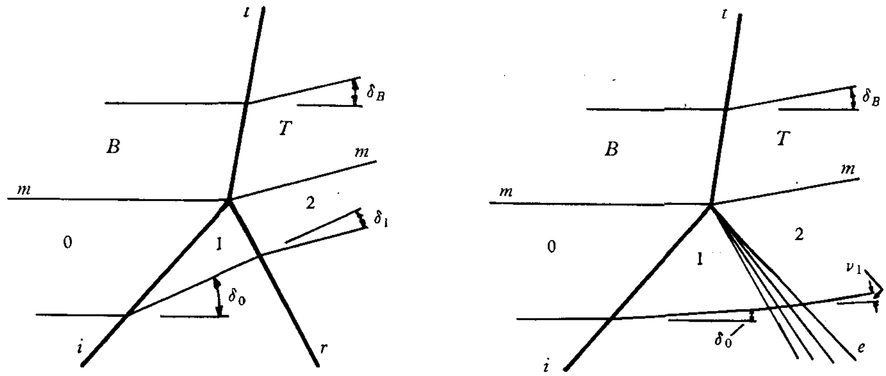


FIGURE 1. Regular wave system nomenclature. *i*, incident shock; *t*, transmitted shock; *r*, reflected shock; *e*, reflected expansion; *mm*, interface.

is given by $a_{0,B} = [\gamma_{0,B} R T_{0,B}/\mathscr{M}_{0,B}]^{\frac{1}{2}}$ but since $T_0 = T_B$ the following relation can be obtained between the Mach numbers upstream of the incident and transmitted shocks:

$$M_B = M_0 \left[\frac{\gamma_0 \mathscr{M}_B}{\gamma_B \mathscr{M}_0}\right]^{\frac{1}{2}}. \tag{7}$$

Equations (1) to (7) will be collectively called the (integrated) equations of motion. There are eight of them altogether and they involve thirteen variables, namely

$$(\gamma_{0,B}, \mathscr{M}_B/\mathscr{M}_0, M_{0,1,2,B}, P_{1,2,T}/P_{0,1,B}, \delta_{0,B}, \nu_1).$$

Thus to define a solution or set of solutions it is necessary to assign values to five of the variables. For this purpose it will be convenient to select $\gamma_{0,B}$, $\mathscr{M}_B/\mathscr{M}_0$, M_0, P_1/P_0. A particular set of values of these variables will be the initial conditions that specify a set of solutions. Because of the transcendental nature of the equations it is not possible to reduce them to a single polynomial as was done previously where the reflected wave was a shock. Numerical results were obtained from the equations by a process that amounted to setting up part of the hodograph diagram on a computer. Solutions were defined by the intersection(s) of the characteristic that represented the expansion, with the polar that represented the transmitted shock. The intersection(s) were found by a simple iterative procedure. Detailed data were compiled for the air–CH_4 and air–CO_2 combinations and as in the earlier paper the results have been presented graphically by plotting the pressure ratio across the transmitted shock against the pressure ratio across the incident shock for a constant Mach number M_0, figures 2 and 3. Additional data needed for special aspects of the investigation are shown in figures 4, 5 and 6. The gas constants were as shown in table 1 and with these values (7) becomes

$$M_{CH_4, CO_2} = 0{\cdot}7711,\ 1{\cdot}284_5 M_{\text{air}}. \tag{8}$$

	Air	CH_4	CO_2
Ratio specific heats γ	1·402	1·303	1·288
Molecular weight $\mathscr{M}$	29·02	16·04	44·01

TABLE 1

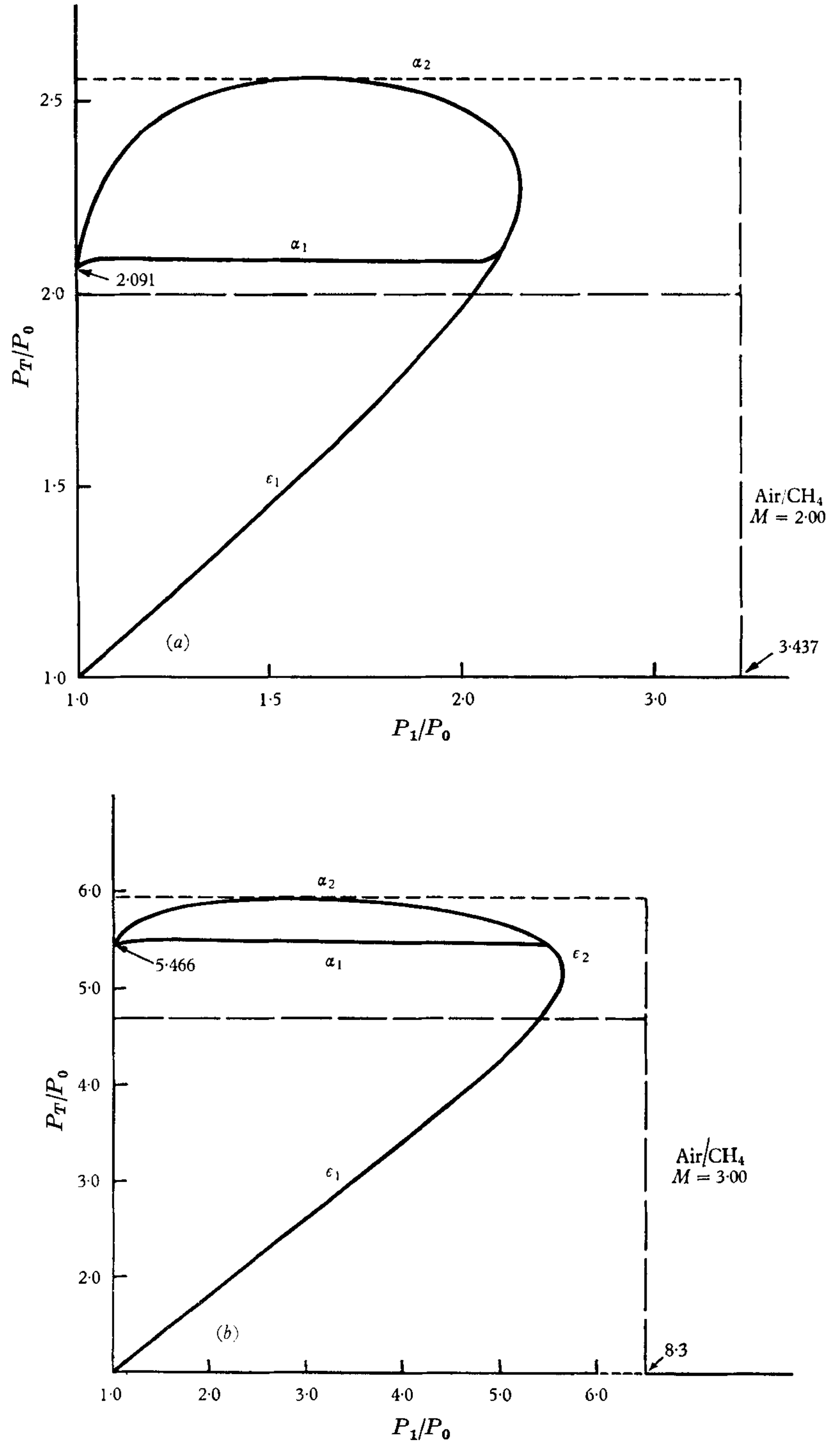
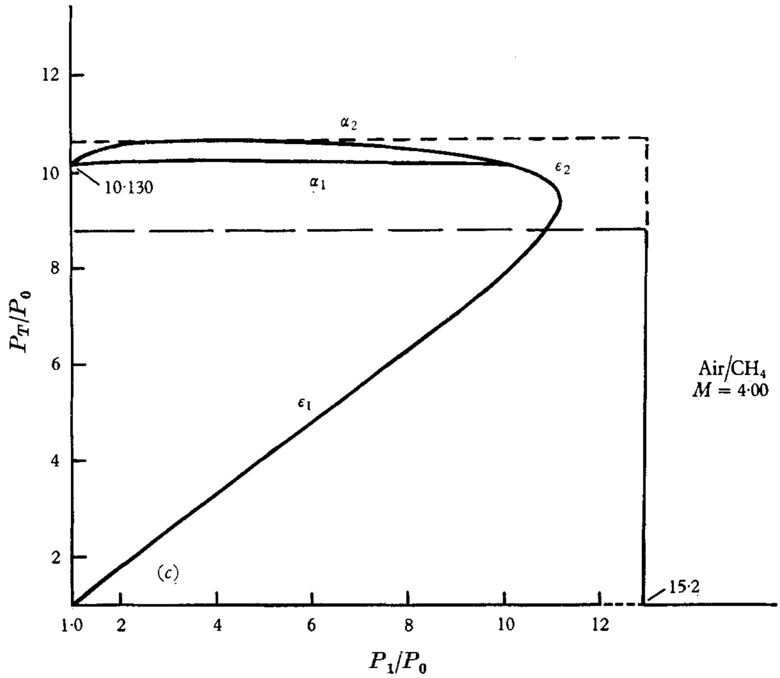


FIGURE 2. Physically significant solutions for the regular refraction of a plane shock at the air–CH_4 interface. ——, solution curve; — —, sonic lines; ---, normal shock line; (a) $M_0 = 2{\cdot}0$; (b) $M_0 = 3{\cdot}0$; (c) $M_0 = 4{\cdot}0$; (d) $M_0 = 5{\cdot}0$.



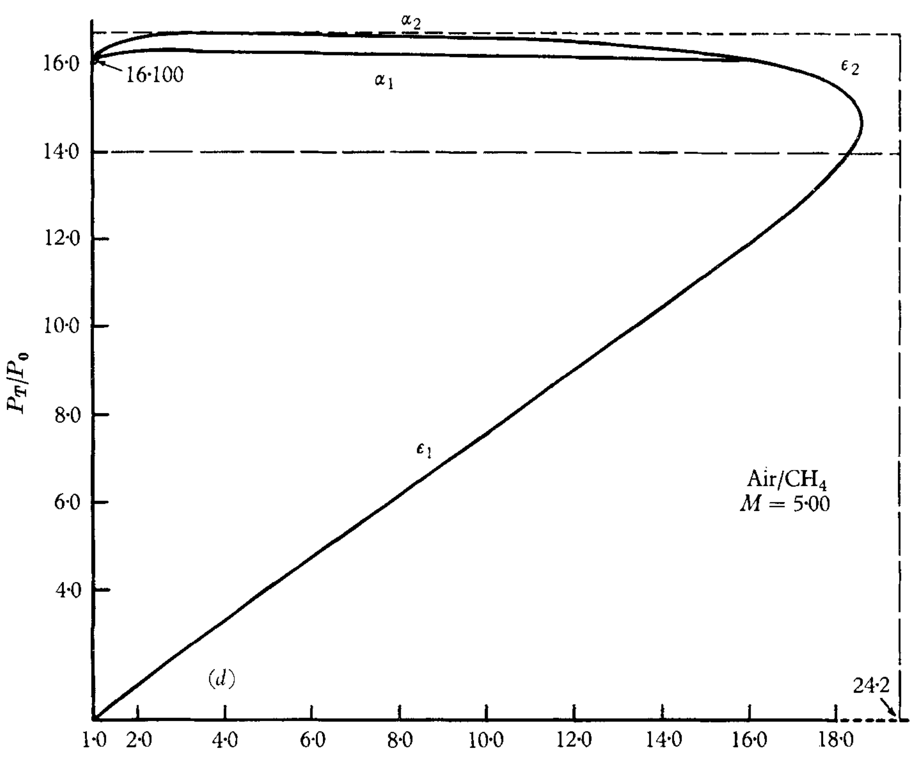


FIGURE 2 (*cont.*)

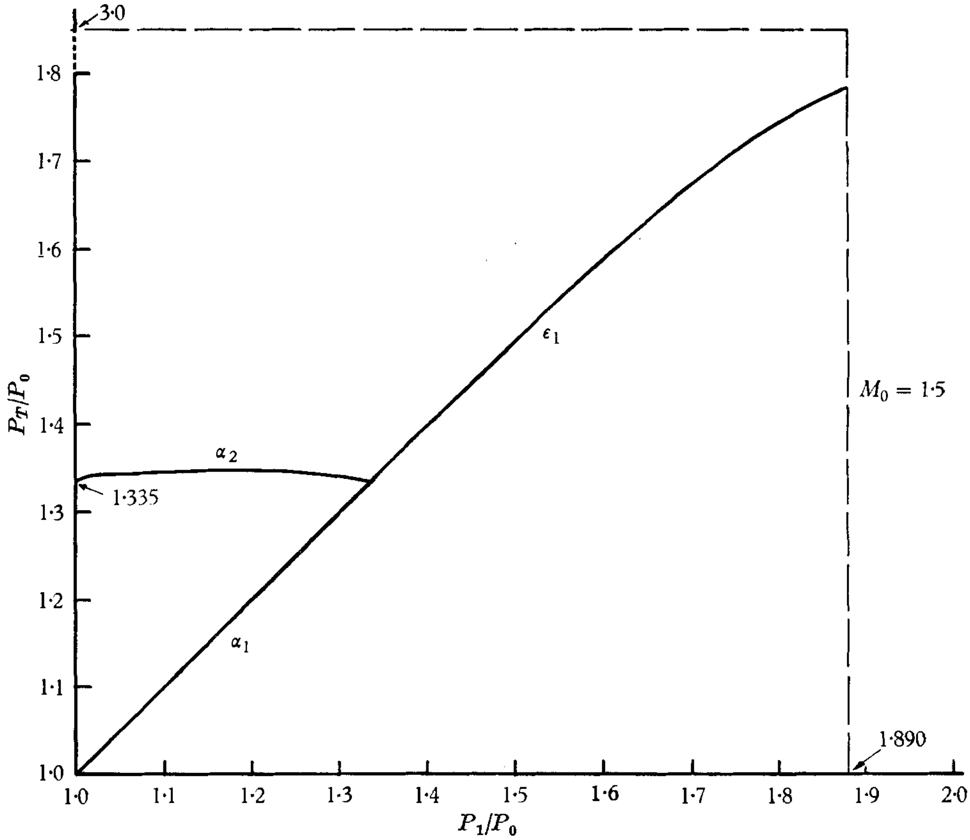


FIGURE 3. Physically significant solutions for the regular refraction of a plane shock at the air–CO_2 interface. ——, solution curve; — —, sonic line; ---, normal shock line; $M_0 = 1{\cdot}5$.

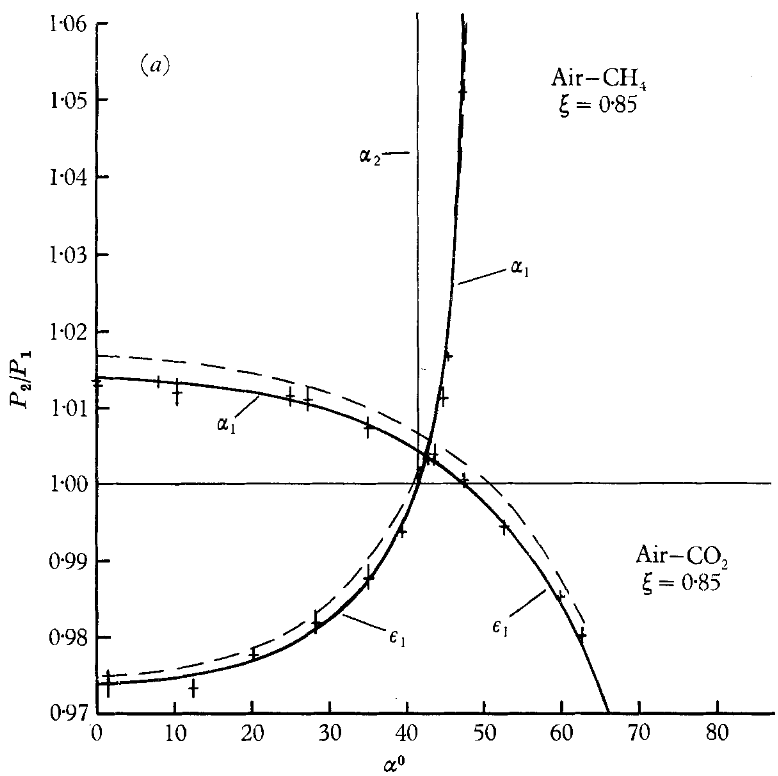


FIGURE 4. Comparison of theoretical and experimental data for regular shock refraction. ——, present theory; ---, Polacheck & Seeger (1951) theory; +, Jahn's (1956) experimental data.

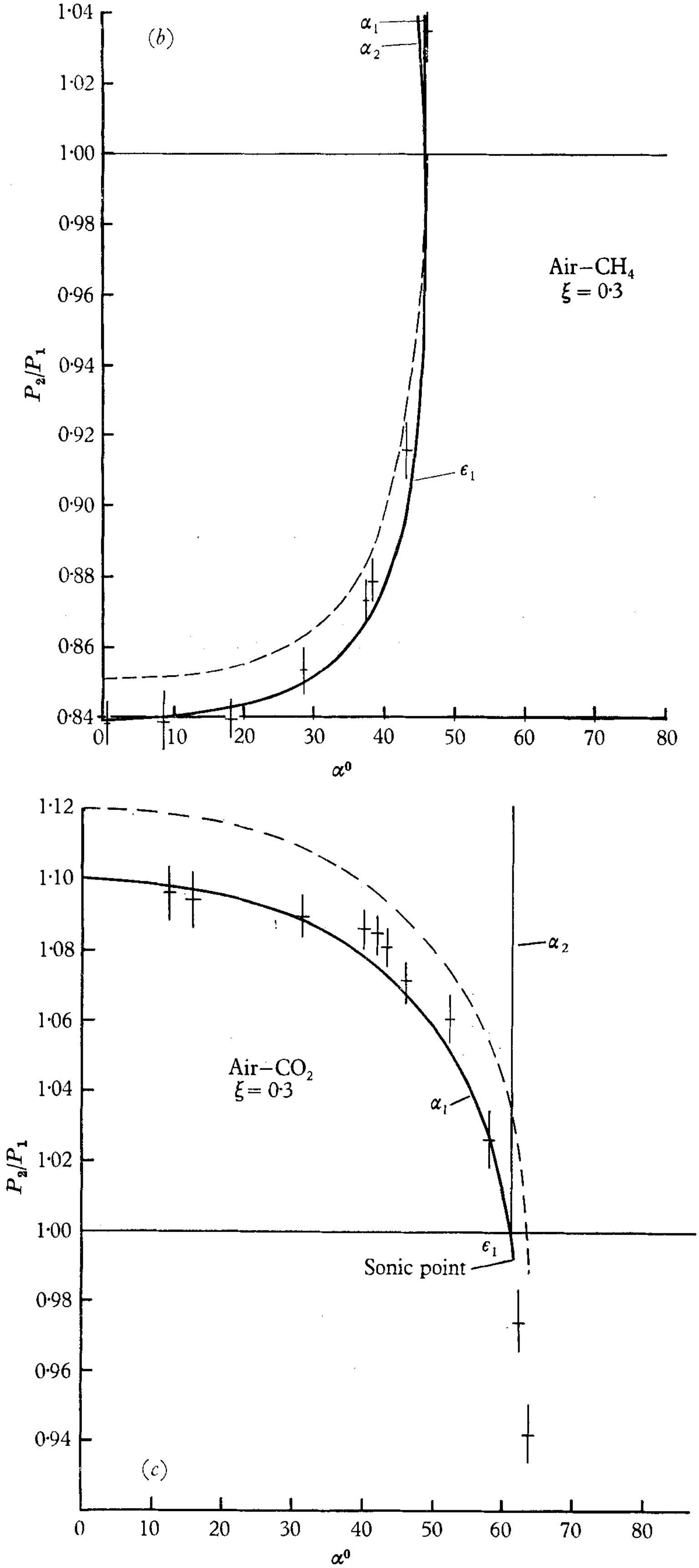


FIGURE 4 (*cont.*)

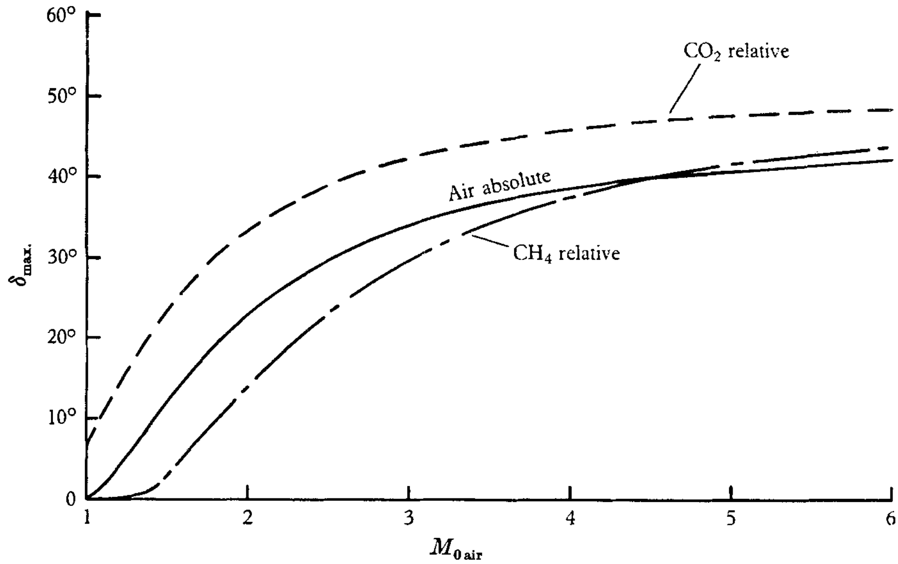


FIGURE 5. Maximum shock wave deflection angles. Absolute value for air; relative value for CH_4, CO_2, that is, $M_{CH_4, CO_2} = 0{\cdot}7711,\ 1{\cdot}284_5 M_0$.

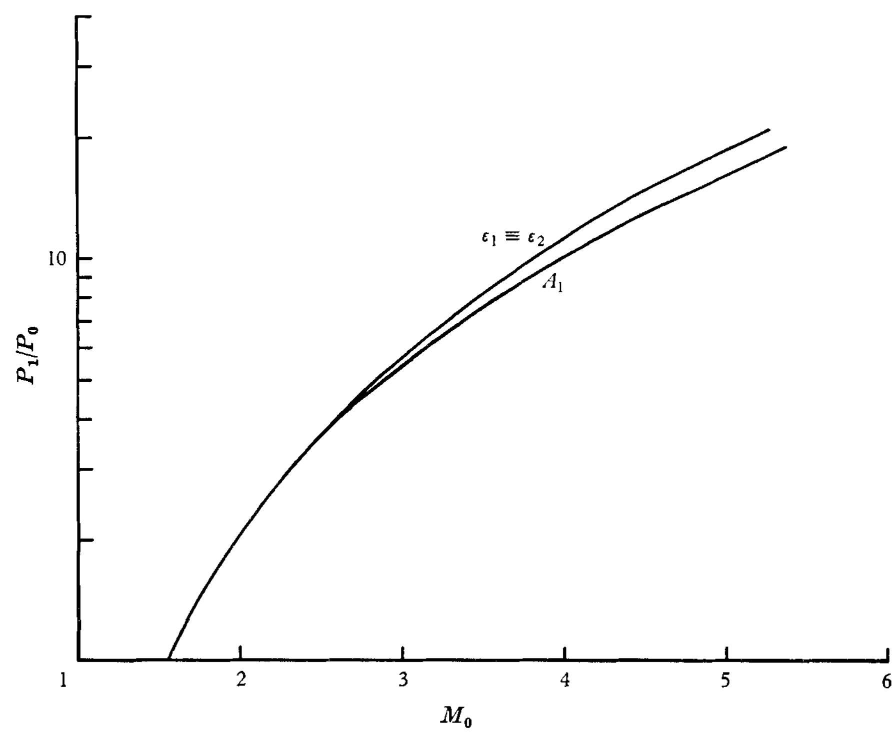


FIGURE 6. Initial conditions for the ordered set (ϵ_1, ϵ_2).

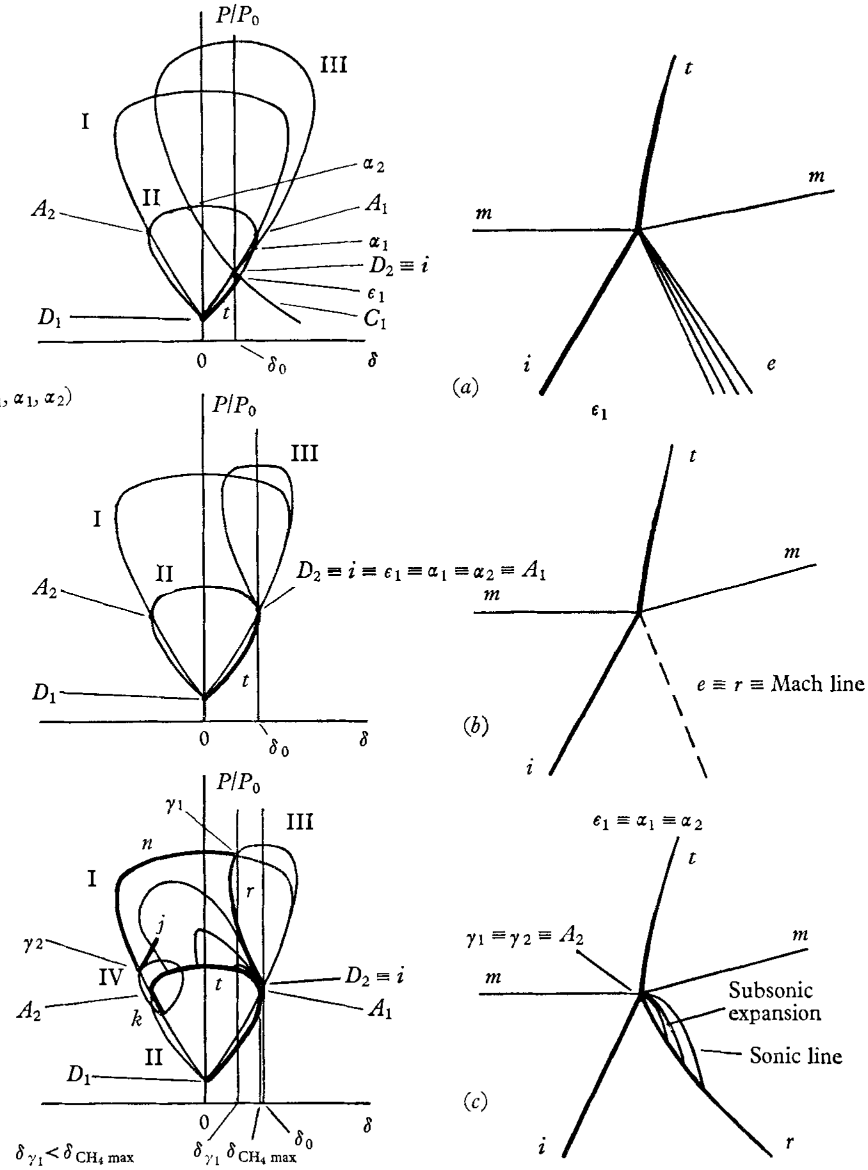


FIGURE 7. Sequence of events obtained by permitting the incident shock i to increase its strength until the refraction becomes irregular. Air–CH_4 interface with initial conditions such that $\delta_{CH_4 \max} < \delta_{\gamma_1}$.

3. The regular refraction at the air–CH_4 interface

For this combination of gases $a_{air} < a_{CH_4}$ and the refraction is said to be 'slow–fast'. In the 1966 paper the hodograph mapping technique was used to construct orderly sequences of events of the various wave phenomena. A particular sequence was built up by imagining that the incident shock was initially a Mach line and then allowing its strength to gradually increase until it approached that of a normal shock. Maps were constructed in both physical and hodograph planes

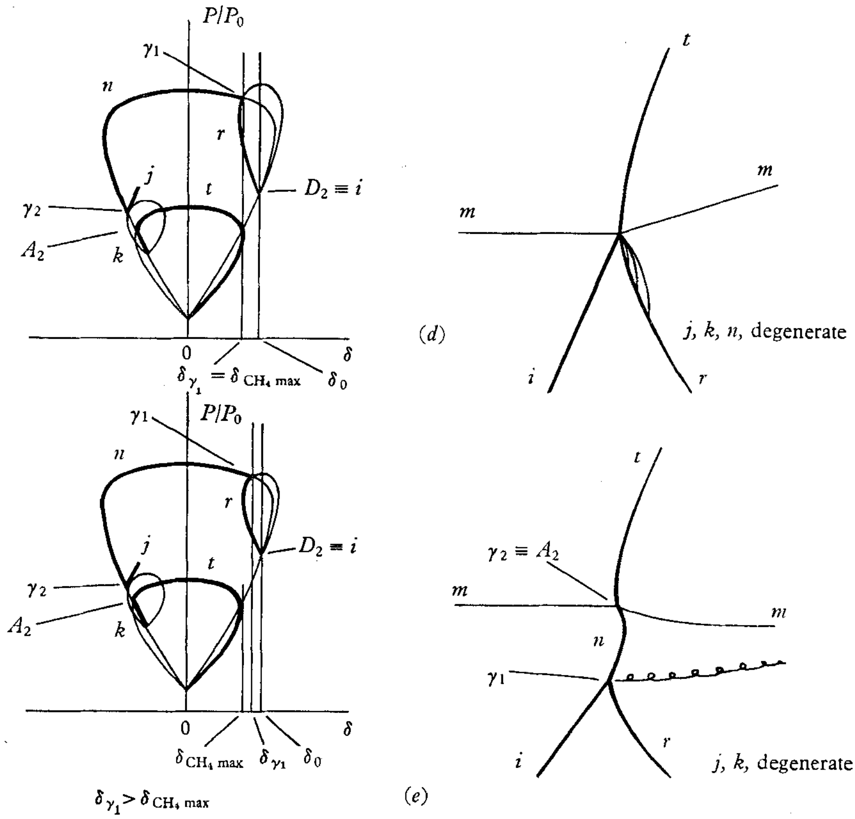


FIGURE 7 (*cont.*)

and by using one map to correct the other a fairly complete qualitative picture of the sequence could be deduced. A typical sequence is shown in figure 7. Further studies on the computer have shown that a hitherto unreported effect exists when $M_0 > 4{\cdot}5$. This is basically due to the maximum shock wave deflexion angle in methane $\delta_{CH_4\,max}$ becoming relatively larger than that in air, $\delta_{CH_4 max} > \delta_{air\,max}$, as M_0 increases, figure 5. Here 'relative' means that $M_{0(air)}$ and M_{CH_4} satisfy (8). The way in which the effect modifies the sequence is shown in figure 8 and it will now be considered in more detail.

With a slight displacement of D_2 the intersection of the characteristic c_1 with polar II defines the solution ϵ_1 which physically represents a regular refraction with a reflected expansion. The displacement also gives the solutions α_1, α_2 which are defined by the intersections of polars II and III. Each of these solutions represents a regular refraction with a reflected shock. The ordered set is thus $(\epsilon_1, \alpha_1, \alpha_2)$. There is no change in the character of the set until the following coincidence is formed, $D_2 \equiv \epsilon_2 \equiv \alpha_1 \equiv \alpha_2 \equiv A_1$. At this condition the reflected waves of $\epsilon_2, \alpha_1, \alpha_2$ degenerate to Mach lines—called a Mach line degeneracy but the reflected wave of ϵ_1 remains of finite strength. For displacement beyond the coincidence there are no intersections of physical interest between polars II and III and the α_1, α_2 solutions cease to exist. Instead the characteristic c_1 now inter-

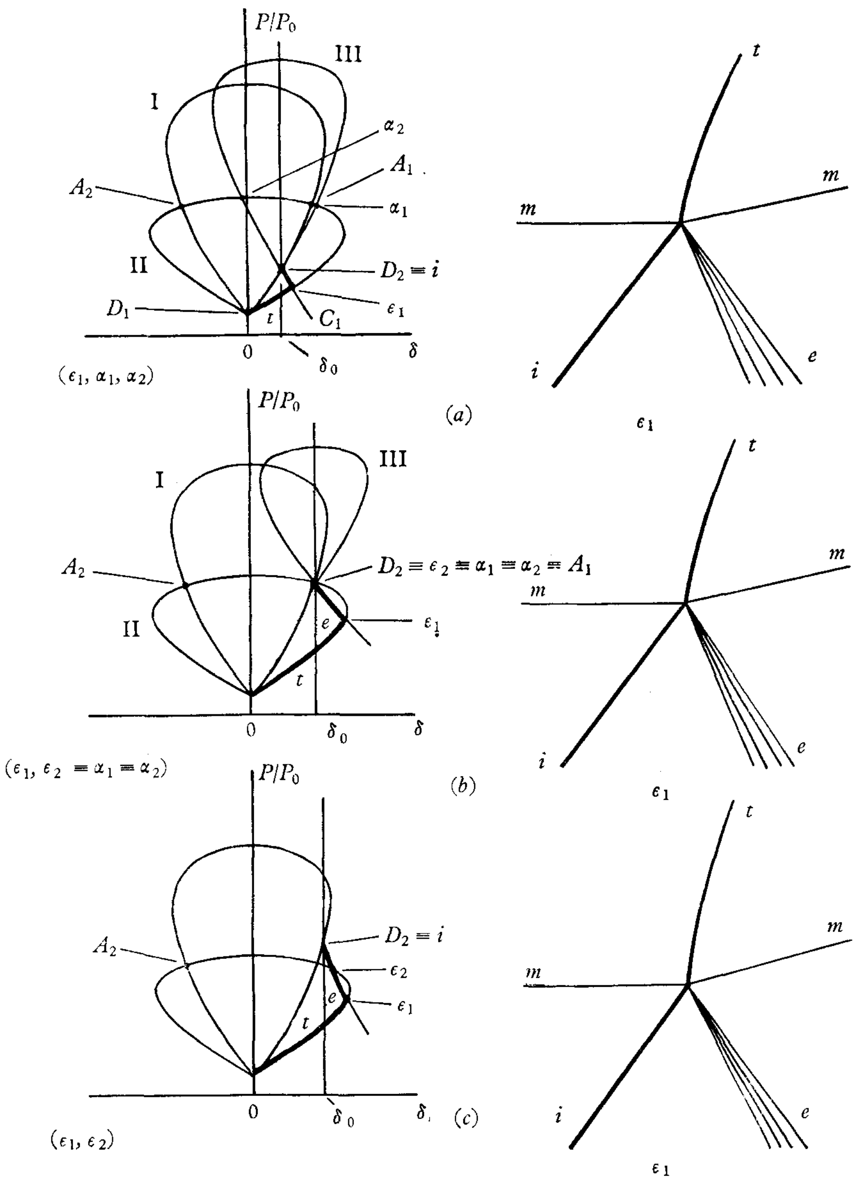


FIGURE 8. Sequence of events obtained by permitting the incident shock i to increase its strength until the refraction becomes irregular. Air–CH_4 interface with initial conditions such that $\delta_{CH_4 \max} > \delta_{\gamma_1}$.

sects polar II in two places and the set becomes (ϵ_1, ϵ_2). Physically both of these solutions represent regular refractions with reflected expansions. The next event of interest is the coincidence $\epsilon_1 \equiv \epsilon_2$ and for displacement beyond this condition the set is empty. It is therefore concluded that $\epsilon_1 \equiv \epsilon_2$ is a boundary between a regular refraction with a reflected expansion and an irregular refraction.

The set $(\epsilon_1, \alpha_1, \alpha_2)$ exists over the range of initial conditions defined by the end-

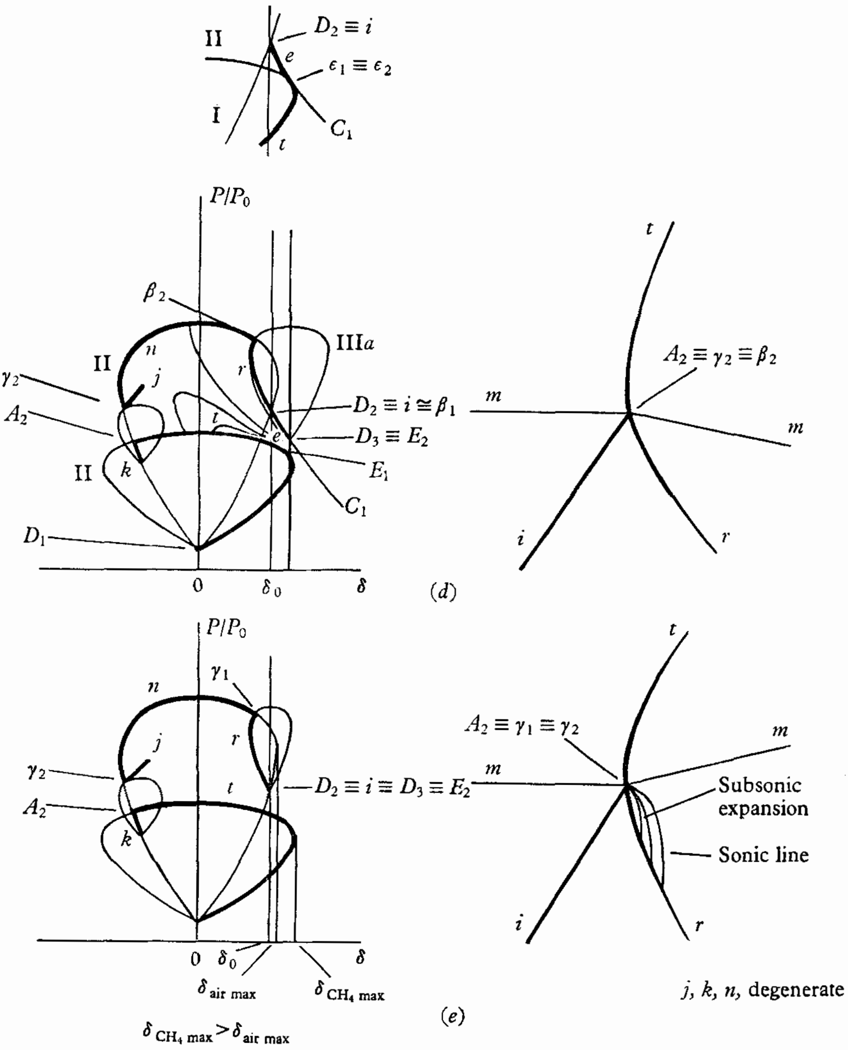


FIGURE 8 (*cont.*)

points $D_2 \equiv D_1$ and $D_2 \equiv A_1$. Physically these coincidences correspond respectively to the incident shock and the reflected wave becoming Mach line degeneracies. The set (ϵ_1, ϵ_2) exists over the range $D_2 \equiv A_1$ to $\epsilon_1 \equiv \epsilon_2$. At the latter condition there is in general no degeneracy so all waves remain at finite strength. The numerical data are shown in figure 2 and for completeness the α_1, α_2 data which have been published before Henderson (1966) are also shown. Further data were obtained in a form that was suitable for comparison with the experimental results of Jahn (1956), figure 4. Jahn compared his data with some early theoretical calculations due to Polachek & Seeger (1951), and their results are also shown. They used idealized values for the gas constants, for example $\gamma_{CO_2, CH_4} = \frac{4}{3}$, and this has a marked effect on the curves as may be seen in figure 4. Jahn used an approximation to correct for this effect. It will be noted that it is the weakest member of the set that agrees with the experimental data. In general, agreement is within the limits of experimental error although some small discrepancies are apparent. In the earlier papers argument was presented to show that the weakest member of the set should also be preferred on theoretical grounds. The conclusion was subject to the provisions that there were no boundaries or strong disturbances in the flow, for if they are present then they may be able to maintain the higher downstream pressures associated with the stronger solutions. In this event one of the stronger solutions may be forced to appear. The process has actually been demonstrated in a classic experiment reported by Busemann (1949).

Consider now the set (ϵ_1, ϵ_2) which has not been discussed previously. The following continuity argument makes it plausible to prefer ϵ_1 subject to the above qualification. Refer to figure 8 and suppose that $D_2 \equiv A_1$, and let the pressure ratio P_0/P_1 across i suffer a small variation. When the increment is negative the set will be $(\epsilon_1, \alpha_1, \alpha_2)$ and there is no doubt that the ϵ_1 solution will appear. When the increment is positive the set will be (ϵ_1, ϵ_2) and the only member common to both sets will be ϵ_1. It is evident that if ϵ_1 also appears in the second case then the transition across the coincidence will be continuous. A small change in the strength of i will then be matched by a small change in the strength of t and the reflected wave e will be of negligible strength. The numerical data show that the process is very nearly linear. By contrast the appearance of the ϵ_2 solution would require that a pressure increment (disturbance) across i, no matter how small, would cause a sudden and finite increase in the strengths of t and e. One way in which the transition $\epsilon_1 \rightarrow \epsilon_2$ could be made plausible is that if simultaneously with crossing the coincidence $D_2 \equiv A_1$ a suitably shaped boundary is brought from infinity and placed behind and close to t. Similarly in the reverse process $\epsilon_1 \leftarrow \epsilon_2$ the boundary would have to be removed to infinity. Thus the discontinuous transition $\epsilon_1 \leftrightarrows \epsilon_2$ is apparently associated with movable boundaries and this suggests that such transitions may occur during oscillatory phenomena involving shock waves. The ϵ_2 solution would also be the plausible choice if the boundary were placed in position *before* the transition so that the α_1 solution was forced to appear. The $\alpha_1 \leftrightarrows \epsilon_2$ transition is also continuous and physically a small variation in the strength of i would be matched by a small variation in the strength of t. The reflected wave is again of negligible strength and here also the numerical results show the variation to be very nearly linear. After studying the remaining transitions one is led to the following hypothesis. *The continuous transitions are* $\epsilon_1 \rightleftarrows \epsilon_1$, $\alpha_1 \rightleftarrows \epsilon_2$, $\alpha_2 \rightleftarrows \epsilon_2$, *and these are associated with steady boundaries; the discontinuous transitions are* $\epsilon_1 \rightleftarrows \epsilon_2$, $\alpha_1 \rightleftarrows \epsilon_1$, $\alpha_2 \rightleftarrows \epsilon_1$, *and these are associated with movable boundaries.*

4. The irregular refraction at the air–CH_4 interface

Beyond the coincidence $\epsilon_1 \equiv \epsilon_2$ the characteristic c_1 no longer intersects polar II and a small gap E_1E_2 opens between them; E_1, E_2 are the former tangency points on both curves. For an infinitesimal gap the state of the gas immediately downstream of the wave system should be largely dominated by conditions at E_1, E_2. This is due to the streamlines crowding together as they pass out through the gap (Busemann 1949; Guderley 1947; Kawamura & Saito 1956; Henderson 1966). The hodograph diagram now begins to resemble the ones shown in figure 7*c*, *d*, *e*, and as a preliminary the salient features of this sequence will be mentioned briefly. Physically the opening of the gap implies that t can no longer provide enough pressure to match the combined pressure across both i and r. It is this *strong* disturbance that forces a basic change in the wave system structure. The hodograph diagram shows that a Mach stem n should be present but it was not detected by Jahn. He found instead a 3 shock system as illustrated in figure 7*c*, *d*. Since, however, the hodograph diagram requires n to be present it is concluded that n is confined to a region that is too small to detect. This will be called a geometric degeneracy. Sternberg (1959) encountered a similar situation in his study of the Guderley wave confluence. It appears from his results that the Guderley patch is too small to be detected in a laboratory scale experiment. With a continued increase in the strength of i the maximum shock attachment angle for methane $\delta_{CH_4\,max}$ is eventually exceeded, $\delta_{\gamma_1} > \delta_{CH_4\,max}$, and this forces n to grow. Growth takes place by r moving away from the interface and along i. In summary it may be said that the irregular wave system shown in figure 7*c* appears when equation (5) can no longer be satisfied while the system shown in figure 7*e* appears when the shock wave analogue of equation (6), namely $\delta_B = \delta_0 + \delta_1$, can no longer be satisfied. The whole process has been discussed in greater detail in our earlier paper. The similarities of the hodograph diagrams in figures 7, 8 suggest that the physics are also similar but there are some important differences and on taking them into account the following picture emerges of the physical plane in figure 8.

When the gap opens the polar map of t is extended around to the intersection A_2 and because A_2 is in the negative δ half-plane this will mean that t must incline forward near the interface. Furthermore, the interface itself will now be deflected downwards instead of upwards. This alters the nature of the 'corner' at the refraction point and requires the reflected wave to change from the expansion e to the shock r. Assuming that these changes take place continuously it will be convenient to suppose that r appears downstream of e and then strengthens while e weakens. An extra polar III*a* is now added to the diagram so that r can be mapped. Its construction commences at $D_3 \equiv E_2$, which represents the state of the gas immediately downstream of e. In general III*a* will make two intersections (β_1, β_2) with polar I and physically both points represent a confluence of three shock waves. As the strength of i continues to increase the gap widens and $D_3 \equiv E_2$ approaches D_2 and finally joins it $D_3 \equiv E_2 \equiv D_2$. In the process e degenerates to a Mach line and β_1 becomes physically trivial. The β_2 solution is now unique† and it is also one end-point of the Mach stem n. The other end-point is near A_2 and it is determined by polar IV. The existence of IV depends on A_2 being on the supersonic part of polar I. The hodograph diagram of figure 8*e* now has some resemblance to the one in figure 7*c* and III*a* may be conveniently relabelled III and β_2 relabelled γ_1.

† The β_2 solution is also preferred in the earlier development because the gap opening is physically equivalent to a strong disturbance in the flow; that is equation (5) is no longer satisfied.

Consider finally the growth of n. It has been noted that this takes place by r being displaced along i. The regular–irregular transition illustrated in figure 8 probably takes place quite quickly. This is due to the substantial changes that are required at the corner once the gap begins to open and even an infinitesimal movement of r will sweep e out of existence. On the other hand it can now be shown that a large displacement of r is not always possible. Thus it will be recalled that the condition for the displacement of r is that $\delta_{\gamma_1} > \delta_{CH_4\,max}$. Clearly this condition will be impossible if $\delta_{CH_4\,max} > \delta_{air\,max}$ but it will be seen from figure 5 that this is just what happens when $M_0 > 4{\cdot}5$. It is therefore concluded that n cannot grow if $M_0 > 4{\cdot}5$ but that it can grow if $M_0 < 4{\cdot}5$. More precisely the boundary between the two conditions is $\delta^*_{\gamma_1} = \delta_{CH_4\,max}$ but the corresponding Mach number is still very nearly $M_0 = 4{\cdot}5$. Figure 7 illustrates a sequence where n does grow and figure 8 where it does not. Accordingly the theory predicts that an experiment performed at the initial conditions corresponding to figure 8 will show that the wave system will change from a regular 3 wave refraction containing a reflected expansion of finite strength to an irregular 3 wave refraction containing a reflected shock of finite strength. Jahn did not report an experiment at the required initial conditions and it probably has not yet been observed.

A further remark is needed on the nature of the corner at the refraction point. During the regular–irregular transition illustrated in figure 8 the Mach stem n gradually comes into existence—although it remains geometrically degenerate. The hodograph map of n lies partly in the negative δ half-plane and partly in the positive δ half-plane and it provides the necessary adjustment in flow direction between i and r on the one hand and t on the other. The problem is to decide which direction prevails in the downstream flow. Owing to the degeneracy of n the maps are of no help. However, Jahn's experiments corresponding to the sequence shown in figure 7 show that the flow is deflected in the positive direction, and it has been assumed that the same thing happens for the wave systems shown in figure 8e where n has become fully formed.

5. The regular refraction at the air–CO_2 interface

For this interface solutions of the ϵ_1 type occur only for a very restricted range of initial conditions. Numerical data are shown in figure 3 and a typical sequence in figure 9. The set is initially† (α_1, α_2) and subsequently changes to (ϵ_1) after the coincidence $D_2 \equiv \epsilon_1 \equiv \alpha_1 \equiv \alpha_2 \equiv A_1$. Additional data are compared with Jahn's experimental results in figure 4. Agreement is good for $\xi = P_0/P_1 = 0{\cdot}85$ but there are no experiments to compare with ϵ_1 when $\xi = 0{\cdot}3$. Our calculations show that the single experimental point obtained by Jahn for this shock strength actually corresponds to an irregular refraction. The regular–irregular transition occurs at $D_2 \equiv S_1$, which corresponds to sonic flow downstream of i. From Jahn's results it is found that the ϵ_1 solution becomes irregular by the expansion fan broadening into a continuous band of expansion waves as illustrated in figure 9. A detailed study of the hodograph diagram shows that in this case it is necessary

† There may exist the set $(\alpha_1, \beta_1, \beta_2, \alpha_2)$ as $P_1/P_0 \rightarrow 1$ but the range of initial conditions is very small (Henderson 1966).

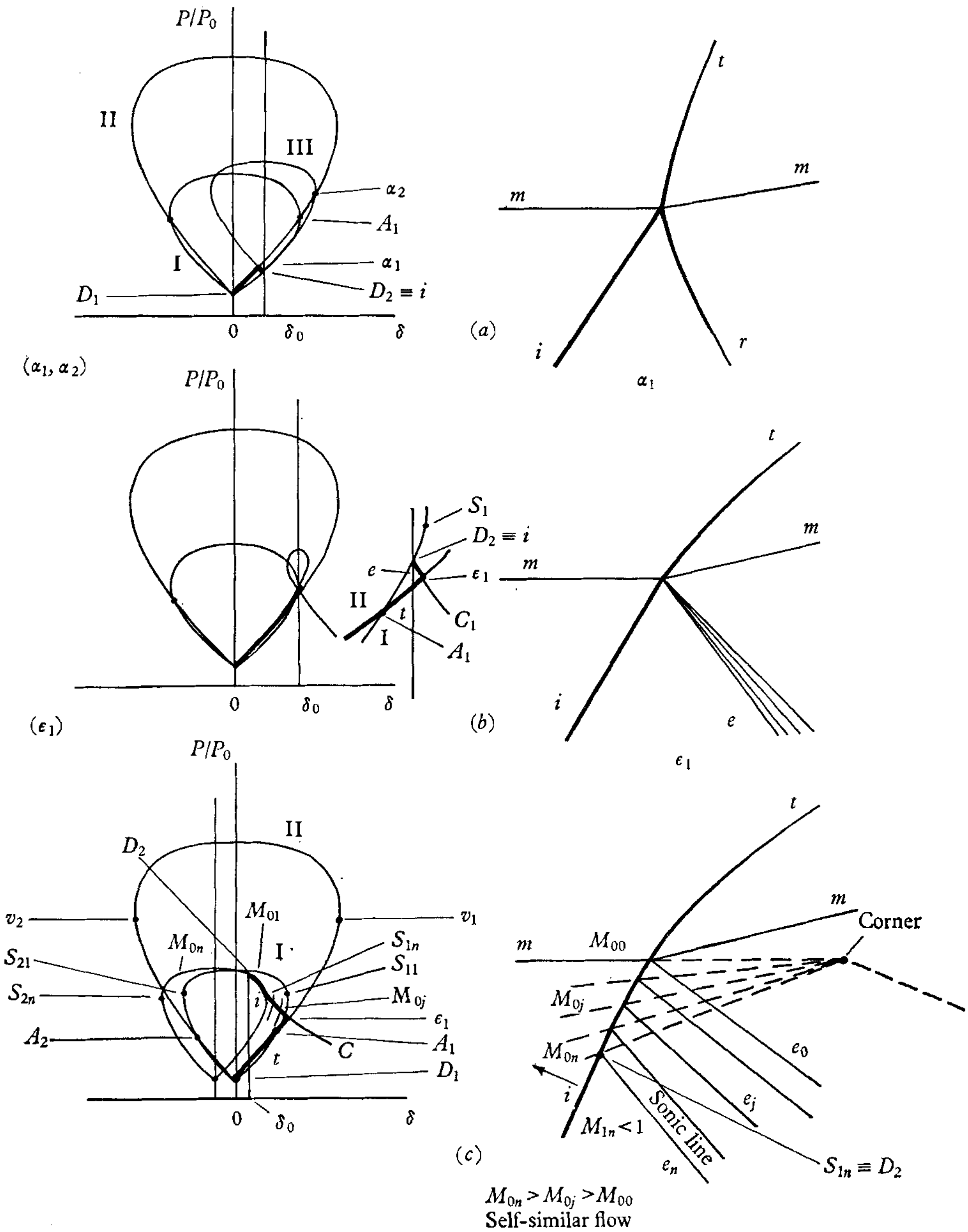
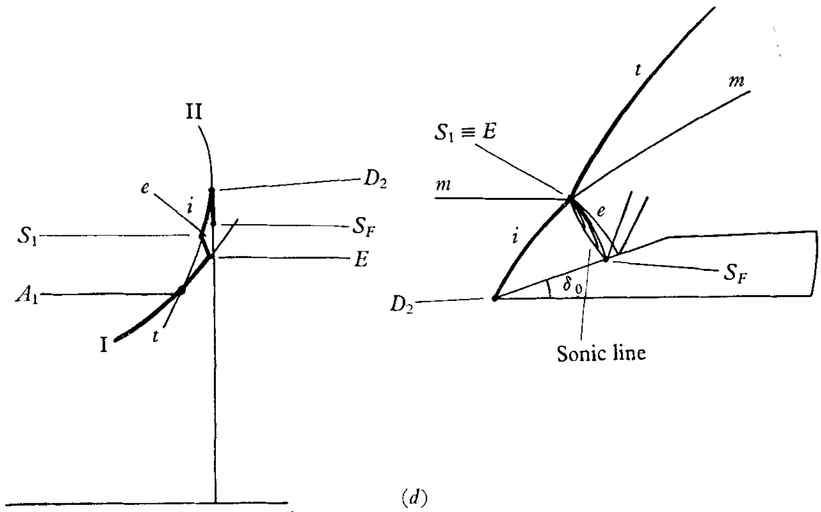


FIGURE 9. Sequence of events obtained by permitting the incident shock i to increase its strength until the refraction becomes irregular. Air–CO_2 interface.

to distinguish between self-similar and steady-state flows. Since Jahn's experiments were performed in a shock tube the flow was substantially self-similar and its solution will be considered first, figure 9*c*. The Mach number M_{0j} is variable upstream of the curved part of i and in particular it decreases monotonically along i as the interface is approached. As a result, i begins to refract *before* it



(*d*)
Steady flow

FIGURE 9 (*cont.*)

reaches the interface. This refraction can be analysed by supposing that the Mach number changes in small discrete steps; then as i encounters each increment it suffers a regular refraction which causes a reflected expansion wave of small intensity to be emitted. In the hodograph plane both the curved part of i and the band of expansion waves map with the same segment and the map terminates where the segment intersects polar II. A somewhat similar flow has been described elsewhere (Henderson 1967) in which under prescribed initial conditions an incident shock emits a band of expansion waves as it penetrates the boundary layer on a flat plate. For steady flow the Mach number is constant upstream of i and therefore i must map into a segment of a single polar and not cut across the polars as in the self-similar solution. The flow also depends on the type of boundary that generates i. In figure 9*d* this is assumed to be a finite wedge. The sonic line begins at the refraction point and ends on the wedge. Instead of a continuous band of expansion waves there is now a centred fan. Guderley (1962, p. 145) has discussed a somewhat similar flow in connexion with the structure of wave patterns in free jets. In summary the hodograph diagram predicts for this irregular refraction that the reflected wave develops into a continuous band of expansion waves when the flow is self-similar but the reflected wave degenerates into a centred expansion fan when the flow is steady.

6. Remarks on the linear character of some solutions

Referring again to figures 2, 3, it will be noted that, for the weakest member of the set, the pressure ratio across the transmitted shock is very nearly proportional to the pressure ratio across the incident shock. Now because the curve passes through the origin it follows from (5) that the constant of proportionality is the pressure ratio across the reflected wave. The weakest member of the set has the same linear character in all the cases that have been computed, and the hodograph diagram indicates that this is a general property of the set. The reflected wave is usually weak; a striking example of this is shown in figure 3 where the pressure ratios across i and t are almost the same. Thus when $P_1/P_0 = 1{\cdot}300$ the computer shows that $P_B/P_0 = 1{\cdot}301$ and hence for the reflected wave $P_2/P_1 = 1{\cdot}001$, which is a very weak wave. Hence even though there is the drastic change in the medium air $\rightarrow CO_2$ there is practically no effect on the strength of the propagating shock† $P_1/P_0 \approx P_B/P_0$. Although other members of the set do result in significant changes in the wave strength as the medium changes they also exhibit an interesting linearity expecially for the slow–fast refraction. This is illustrated by the α_1 solution in figure 2 where over much of the range of initial conditions the strength of t is almost *independent* of the strength of i. It follows from (5) that the strengths of i and r are almost inversely proportional. The effect is most marked at higher M_0 where the α_2 solution also begins to display it.

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† Of course there may be marked changes in the other wave properties such as the wave angle.